

Solving Systems of Equations

Solving a system of equations by SUBSTITUTION with “y =” or “x =” equation

Given a system of equations:

$$y = 2x - 6$$

$$y = 9 - x$$

Combine the equations by replacing the ‘y’ with ‘2x - 6’ in the equation $y = 9 - x$.

solve the new equation

$$2x - 6 = 9 - x$$

$$2x + x = 9 + 6$$

$$3x = 15$$

$$x = 5$$

Back substitute to find y

$$y = 2x - 6$$

$$y = 2(5) - 6$$

$$y = 10 - 6$$

$$y = 4$$

Write the answer

The solution

is (5, 4)

Solving a system of equation using ELIMINATION method

Given a system of equations

$$4x + 2y = 10$$

$$x - 3y = 6$$

multiply by 3

multiply by 2

$$12x + 6y = 30$$

$$\underline{2x - 6y = 12}$$

$$14x = 42$$

$$x = 3$$

Add the equations and solve for the remaining variable

Back Substitute

$$x - 3y = 6$$

$$3 - 3y = 6$$

$$(-3) + 3 - 3y = (-3) + 6$$

$$-3y = 3$$

$$y = -1$$

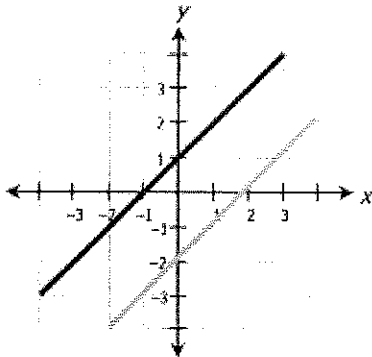
Write the solution

The solution is (3, -1)

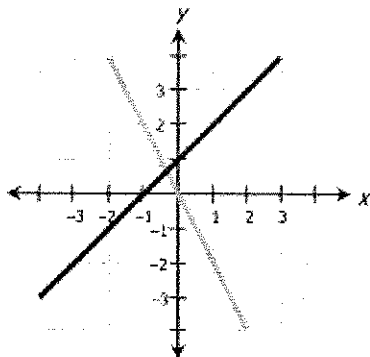
Cut out and put into Notes Notebook

Types of Solutions for Systems of Equations

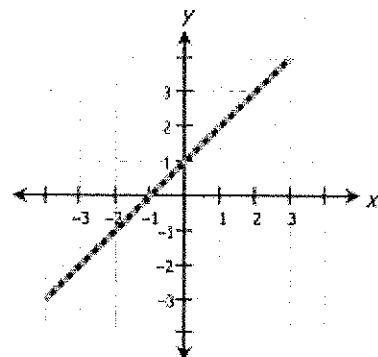
- **One Solution** – When lines intersect at one point, there is only one solution, (x, y) . This solution can be found graphically or algebraically (using substitution or elimination)



No Solution



One Solution



Infinite Solutions

- **No Solutions** – When lines are parallel, there is no common solution. What does this mean algebraically? If we consider the slope-intercept forms of the two lines ($y = mx + b$), it means that the slopes (m) are the same and the y -intercepts (b) are different.
 - When solving for one of the values you will find a FALSE statement like $5 = 3$
- **Infinitely Many Solutions** – When lines overlap, there are infinitely many solutions. Algebraically, this means that the two lines have the same slopes (m) and the same y -intercepts (b).
 - When solving for one of the values you will find a TRUE statement $4 = 4$

Example

$$\begin{aligned} 2x + y &= 5 \\ 4x + 2y &= 10 \end{aligned}$$

How many solutions does the pair of linear equations shown above have? _____

How do you know?

$$\begin{aligned} 2x + y = 5 &\rightarrow 2(2x + y = 5) \\ 4x + 2y = 10 &\rightarrow 4x + 2y = 10 \end{aligned}$$

$$4x + 2y = 10$$

$$4x + 2y = 10$$

Same equations \rightarrow Infinitely many solutions