Exercises *Practicing Habits of Mind*

Check Your Understanding

1. Use the polynomial $3 + 4x^2 - 5x + \frac{3}{4}x^5 - 17x^2 + x^4$.

a. Write the polynomial in normal form.

b. What is the degree of the polynomial?

c. What is the coefficient of x?

d. What is the coefficient of x^2?

e. What is the coefficient of x^3?

f. Find a polynomial that when added to this one gives a sum of degree 3.

2. Describe how each polynomial identity is related to the corresponding number fact.

a. $(x + 5) + (x^2 + 2x + 4) = x^2 + 3x + 9$

$15 + 124 = 139$

b. $(x + 1)(x^2 + 2x + 4) = x^3 + 3x^2 + 6x + 4$

$11 \cdot 124 = 1364$

c. $(x + 1)^2 = x^2 + 2x + 1$

$11^2 = 121$

d. $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$

$11^3 = 1331$

3. a. What is the prime factorization of 120? Of 168?

b. How can you use the prime factorizations to find the greatest common factor of 120 and 168?

c. What is the greatest common factor of $273{,}375 = 3^7 \cdot 5^3$ and $140{,}625 = 3^2 \cdot 5^6$?

d. Suppose x and y are prime numbers. What is the greatest common factor of x^7y^3 and x^2y^6?

e. What is the greatest common factor of $120x^7y^3$ and $168x^2y^6$?

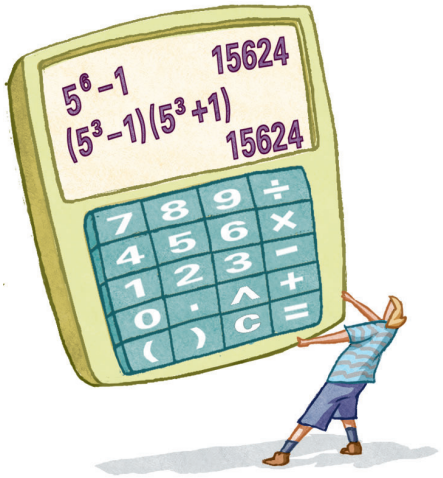
How do you find the greatest common factor for monomials?

4. Transform the expression below into normal form. For what values of a is the coefficient of x equal to 0?

$$(x^2 + 3x + a)(x^2 + 3x - 7)$$

For Exercises 5–10, complete each of the following.

- Calculate three numeric examples that follow from each identity by substituting a number for each variable.
- Prove each identity is true. Use the basic rules of algebra.



5. $x^6 - 1 = (x^3 - 1)(x^3 + 1)$

6. $x^6 - 1 = (x - 1)(x + 1)(x^2 + x + 1)(x^2 - x + 1)$

7. $x^6 - 1 = (x^2 - 1)(x^4 + x^2 + 1)$

8. $x^3 - 1 = (x - 1)(x^2 + x + 1)$

9. $(s + t)^2 - (s - t)^2 = 4st$

10. $(n + 1)^2 - n^2 = 2n + 1$

11. **Take It Further** Show that this equation is an identity.

$$(x^3 - 1)(x^3 + 1) = (x^2 - 1)(x^4 + x^2 + 1)$$

On Your Own

12. Use the identity below. Calculate three numeric examples that follow by substituting for x. Then prove that the identity is true, using the basic rules of algebra.

$$(x^2 - x)(x + 1) = (x^2 + x)(x - 1)$$

13. Here are five equations of polynomials. All the expressions are in normal form. A few of the terms are hidden in each expression.

Three of these cannot be identities. Which three are they? Explain.

A. $3x^3 + \blacksquare + 2x + 1 \stackrel{?}{=} \blacksquare + 2x + 4$

B. $\blacksquare + x^2 - 9 \stackrel{?}{=} x^3 + \blacksquare - 9$

C. $\blacksquare + 3x^2 + \blacksquare + 6 \stackrel{?}{=} x^3 + 3x^2 + \blacksquare$

D. $x^7 + 7x + \blacksquare \stackrel{?}{=} 3x^7 + \blacksquare + 11$

E. $x^2 + \blacksquare + 4 \stackrel{?}{=} \blacksquare + x^2 + \blacksquare$

14. Show that each equation is an identity.

a. $m^2 - n^2 = m(m - n) + n(m - n)$

b. $m(m - n) + n(m - n) = (m + n)(m - n)$

c. $(m + 1)(m - n) + (n - 1)(m - n) = (m + n)(m - n)$

15. **Standardized Test Prep** Which polynomial is NOT equivalent to the other three polynomials?

A. $(3x^3 + 3x^2 + 6x - 1) - 7(x^2 - 1)$

B. $(x^2 + 2)(3x - 4)$

C. $(4x^3 - 4x^2 + 6x - 9) - (x^3 - 1)$

D. $(x - 1)(3x^2 + 5) + (-x^2 + x - 3)$

16. What is the coefficient of the given term in the normal form of $(x + x^2 + x^3 + x^4 + x^5 + x^6)^2$?

a. x^8 **b.** x^{10}

What is the coefficient of the given term in the normal form of $(x + x^2 + x^3 + x^4 + x^5 + x^6)^3$?

c. x^{10} **d.** x^{20}

Maintain Your Skills

17. Find the normal form of each polynomial.

a. $(1 + x + x^2)(1 + x^3)$

b. $(1 + x + x^2 + x^3)(1 + x^4)$

c. $(1 + x + x^2 + x^3 + x^4)(1 + x^5)$

d. Describe a pattern.

18. Expand each power. Replace x with 1. What is your result?

a. $(x - 1)^2$ **b.** $(x - 1)^3$ **c.** $(x - 1)^4$

d. Describe a pattern. Explain why this pattern exists.

19. Write each polynomial in normal form. What is the degree?

a. $x(x + 1)$ **b.** $x(x + 1)(x + 2)$

c. $x(x + 1)(x + 2)(x + 3)$ **d.** $x(x + 1)(x + 2)(x + 3)(x + 4)$

e. Describe a pattern. Explain why this pattern exists.

20. Write each polynomial in normal form. What is the sum of the coefficients of each polynomial?

a. $x(x + 1)$ **b.** $x(x + 1)(x + 2)$

c. $x(x + 1)(x + 2)(x + 3)$ **d.** $x(x + 1)(x + 2)(x + 3)(x + 4)$

e. Describe a pattern.