9. Suppose the sides of a triangle have lengths a, b, and c. A famous formula, proved by Heron of Alexandria in about A.D. 60, states that you can calculate the area of the triangle by this rule.

$$A = \frac{1}{4}\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}$$

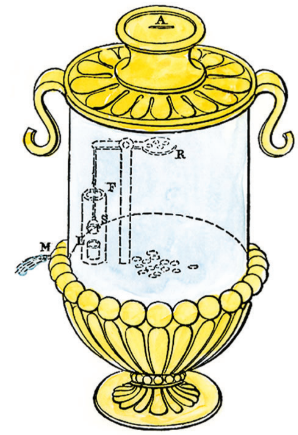
a. Find the area of a triangle with side lengths 13, 14, and 15.

b. **Take It Further** Expand this product.

$(a+b+c)(a+b-c)(c+a-b)(c-a+b)$

c. Use Heron's formula to derive a formula for the area of a triangle with sides all the same length.

d. **What's Wrong Here?** What does the formula give for the area of a triangle with side lengths 6, 4, and 10? Explain.



Heron invented the first vending machine. Can you see how inserting a coin would cause the machine to dispense a small amount of water?

10. **Take It Further** Many geometry books state Heron's formula as follows.

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

Show that this expression is equivalent to the expression in Exercise 9.

On Your Own

11. Without expanding, what is the coefficient of x^4 in the normal form of $(x^2+3x^2+1)(2x^4-x^3+5x+2)+(2x^4-x^3+5x+2)$?

12. The coefficient of x^3 in the normal form of the polynomial below is 18.

$$(x^2+3x^2+1)(2x^4-x^3+5x+2)+(2x^4-x^3+5x+2)$$

What is the coefficient of x^3 in the normal form of $(x^2+3x^2+1)(2x^4-x^3+5x+2)-(2x^4-x^3+5x+2)$?

13. What is the coefficient of x^5 in the normal form of $(x+1)^5$?

14. **Write About It** Revisit what you explored in Lesson 2.05.

a. Explain why the coefficient of x^8 in the normal form of this polynomial is 5.

$(x+x^2+x^3+x^4+x^5+x^6)^2$

b. Explain why the coefficient of x^{10} in the normal form of $(x+x^2+x^3+x^4+x^5+x^6)^2$ is the number of ways you can roll a sum of 10 if you throw two number cubes.

c. Explain why the coefficient of x^{14} in the normal form of $(x+x^2+x^3+x^4+x^5+x^6)^3$ is the number of ways you can roll a sum of 14 if you throw three number cubes.